A fast binary-image comparison method with local-dissimilarity quantification

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Abstract

Image similarity measure is widely used in image processing. For binary images that are not composed of a single shape, a local comparison is interesting but the features are usely poor (color) or difficult to extract (texture, forms). We present a new binary image comparison method that uses a windowed Hausdorff distance in a pixel-adaptive way. It enables to quantify the local dissimilarities and to give their spatial distribution which greatly improve the dissimilarity information. Combined with a Support Vector Machine classifier, this method is successfully tested on an medieval-impression database.

Keywords

Image comparison, binary images, Hausdorff distance, local dissimilarity measure, classification, SVM.

Introduction

Similarity measure is widely used in different domains: image retrieval [8], image classification [2], image quality evaluation [10], registration [1] ... Methods in the literature for similarity measure between images can be classified into two approaches: a) an image feature extraction (shape, curve, texture, histogram) followed by a feature comparison; b) straight image comparison, *i.e.* without feature extraction (*e.g.* SNR, MSE, Hausdorff distance). For the binary images that are not composed of a single shape, the color attribute is poor and sometime object shapes cannot be clearly identified. In this case, the second approach, straight image comparison, seems adapted. Nevertheless, it is interesting to use a local comparison to compensate the lack of shape extraction. Our work presents a new comparison method that evaluates directly and locally the similarity between binary images without feature extraction and any *a priori* knowledge.

In a first part, we define a local Hausdorff measure by using a window and adapting the Hausdorff measure to it. Then, in a second part, the window size is determined according to the local dissimilarity. A mathematical expression of the local measure is given when the window slides over the images to compare. Finally, in a third part, the developed measure is successfully applied in binary images classification.

1 Binary image comparison method

1.1 Dissimilarity measure based on the Hausdorff distance

Among dissimilarity measures over binary images, the Hausdorff distance (HD) has often been used in the content-based retrieval domain and is known to have successful applications in object matching [4] or in face recognition [9]. For finite sets of points, the HD can be defined as [4]:

definition 1 (Hausdorff distance) *Given two non-empty finite sets of points $F = (f_1, \ldots, f_n)$ and $G = (g_1, \ldots, g_m)$ of $\mathbb{R}^2$, and an underlying distance d, the HD is given by*

$$D_H(F,G) = \max\left(h(F,G), h(G,F)\right) \quad (1)$$

$$\textit{where } h(F,G) = \max_{f\in F}\left(\min_{g\in G} d(f,g)\right), \quad (2)$$

$h(F,G)$ is the so-called *directed Hausdorff distance*.

The classical HD has good properties but it measures the most mismatched points between F and G, and as a consequence it is sensitive to noise [6]. Indeed, considering two images containing the same pattern and one point added to the first image, far from the pattern, then the HD will measure the distance between the pattern and the point.

Several modifications of the HD have been proposed to improve it such as: the partial HD [4], the modified HD (MHD) [3], the censored HD [6], the "doubly" modified

Hausdorff distance [9], the least trimmed squared HD [7] and the weighted Hausdorff distance [5]. Those improved HD are detailed in [11].

It is noticeable that except for the MHD, at least one arbitrary parameter has to be determined. The parameter must be chosen to make the measure as discriminating as possible and it depends obviously on the kind of images, and sometimes on the compared images in the same application (*e.g.* more or less dark or noisy images). The MHD matching performance is not as good as the partial HD and the censored HD.

Moreover, these measures are global and cannot account for local dissimilarities. Indeed, the principle of HD is to be a "max min" distance so the value of the HD between two images is reached for at least one couple of points. But it doesn't say if the value is reached in several parts or only for one pair, which corresponds to different degrees of dissimilarity. These remarks motivate us to design a local and parameter-free HD in the next section.

1.2 Definition of the windowed Hausdorff distance

The main reasons of the modification is that the DH is not define for empty sets and this case is possible in a window. Moreover, the obtained measures when the window is sliding or growing must be consistent. A solution is to introduce the distance to the window side as it follows:

definition 2 (Windowed Hausdorff distance) *Let F, G be two bounded sets of $\mathbb{R}^2$. $HD_W(F,G) = \max\left(h_W(F,G), h_W(G,F)\right)$*
where there are three cases

1. $h_W(F,G) =$
$$\max_{f\in F\cap W}\left[\min\left(\min_{g\in G\cap W} d(f,g), \min_{w\in Fr(W)} d(f,w)\right)\right]$$
if $F\cap W \neq \emptyset$ and $G\cap W\neq\emptyset$,
2. $h_W(F,G) = \max_{f\in F\cap W}\left[\min_{w\in Fr(W)} d(f,w)\right]$ *if $F\cap W\neq\emptyset$ and $G\cap W=\emptyset$,*
3. *0 if $F\cap W=\emptyset$,*

where $Fr(W)$ stands for the frontier of the set W.

remark 1
- *In case there is no point of F neither of G in W, both of the directed distances are equal to 0 and therefore the global distance too. This is consistent with the fact that the two extracted parts are equal.*
- *In case there is exactly one set without point in W, one of the two directed distances is equal to 0 and the expression of the other one takes into account the distance to the edge.*

1.3 Window-size choice

The definition of the windowed HD enables to make a local distance but it introduces a parameter which is the window size. It can be chosen by the user, or automatically and globally, or locally according to the local surrounding. The following properties of the windowed HD allow to fix locally the window size and then to evaluate the local dissimilarity.

property 1 (Identity) *Let F, G be two bounded sets of points of $\mathbb{R}^2$, and W a convex closed subset of $\mathbb{R}^2$. $HD_W(F,G)=0 \iff F\cap W = B\cap W$*

The following properties need the window W to be a ball.

property 2 (Boundary) *Let $x\in\mathbb{R}^2$ and $r>0$, and let define $W=B(x,r)$ then $HD_W(F,G)\leq HD(F,G)$.*

This property ensures that the new pieces of information that are taken into account when the window is enlarged do not reduce the former dissimilarity-measure value.

property 3 (growth) *Let $V = B(x_v,r_v)$ and $W = B(x_w,r_w)$ be two close discs such as $V\subset W$ then $HD_V(F,G)\leq HD_W(F,G)$.*

Prop. 2 and 3 ensure that a growing sequence of centered windows gives an increasing and bounded sequence of measure values. It enables to give a stop criterion for such a growing sequence of windows and to be sure it will be satisfied. The measure obtained when the criterion is satisfied is called *local Hausdorff distance* and the set of measures obtained when this measure is computed for all the pixels is called *Local Distance Map* (LDMap). The folowing paragraph presents the algorithm associated to the stop criterion that has been selected.

Algorithm

A practical algorithm for the computation of the local HD map is proposed below (alg. 1). It consists of a sliding window whose radius is locally adapted to be optimal. It shows

Algorithm 1 Computation of LDMap

```
compute D_H(F,G)
for all pixel x do
    n := 1 {initialization of the window-size}
    while HD_{B(x,n)}(F,G) = n and n ≤ HD(F,G) do
        n := n + 1
    end while
    LDMap(x) = HD_{B(x,n-1)}(F,G) = n - 1
end for
```

the way to adapt the window to the local dissimilarity: this

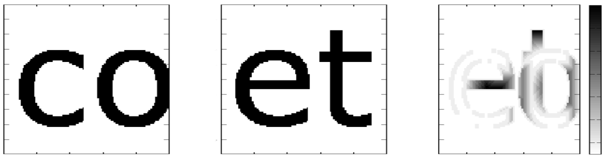
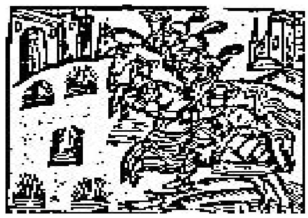


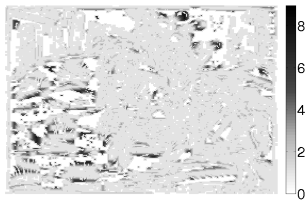
Figure 1. Letters CO et ET and their LDMap illustrating their local dissimilarities

(a) Original impression

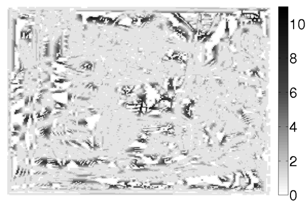


(b) Similar impression

(c) Dissimilar impression



(d) LDMap between 2(a) et 2(b)



(e) LDMap between 2(a) et 2(c)

Figure 2. Medieval impressions and their LDMaps.

step is done in the while loop. Nevertheless, this algorithm is time consuming. Indeed, the computation complexity is in $O(m^4)$ for two $m \times m$ pixel images. The next section presents a formula for the measure that saves a most of the time computation. The computation is faster but the interpretation in terms of local dissimilarity measure comes from algorithm 1.

1.4 Dissimilarity map

property 1 (LDMap mathematical formula)

$$\forall x \in \mathbb{R}^2,$$

$$CDL(x) = |B(x) - A(x)| \max(d(x, A), d(x, B)) \quad (3)$$

The formula gives for each pixel x a value that depends on the distance transform of the sets A and B. Fast algorithms have been developed for distance transformation. Their computation complexity are $O(m^2)$ for $m \times m$ images. So the LDMap complexity with the formula is a $O(m^2)$, which is linear in the pixel number.

Figure 1 illustrates the notion of local dissimilarity. The dissimilarities are quantified: big dissimilarities are represented in dark and small ones in bright. Moreover, they are spatially localized: from the LDMap, one can see that the bigger dissimilarities are situated on the straight line of the ”e” and on the top of the ”t”, so the corresponding dissimilarities are important. The ones between the left part of the ”o” and the bottom of the ”t” and between the loop of the ”e” and the ”c” are bright so they are small.

2 Application

The test database is composed of digitalized medieval illustrations provided by Troyes' library in the framework of a collaboration with the laboratory CReSTIC. These images, originally printed in books, have strong contrast which allows to binarize them with almost no loss. This database is composed of 68 images, some of them illustrating the same scene. The objective is to retrieve illustrations of the same scene.

Fig 2 gives an illustration of medieval impressions and their LDMaps. Fig 2(a) and 2(b) are similar so the higher values are altogether in some parts (standing for the grass and the helmets) of their LDMap (fig 2(d)). Fig 2(a) and 2(c) are dissimilar and the spatial distribution is more randomized (fig 2(e)). The spatial information bring about 5% of efficiency (*cf* tab 1).

One of the difficulties comes from the numerous classes in the database:there are about 30 classes of similar images for the 68 images (some classes contain only one image). The number of classes makes a straight comparison on the images difficult and in addition, the choice of the class number is awkward as far as new images can be introduced and need the creation of a new class. Unlike to the images, the LDMaps are classified into two classes: the ones obtained by the comparison of similar images C_{sim} and the other ones by the comparison of two dissimilar images C_{dissim}. The introduction of news images does not change the LDMap class number.

The comparison of the 68 images results in 2278 LDMaps, 125 of which are classified in C_{sim} and 2153 in C_{dissim} thanks to a manual comparison of the impressions by an expert.

The experiment was carried out by the following way: first a supervised learning is made on a set of 50 LDMaps

Successful retrieval	LDMap	HD	PHD	SVV
found in $\mathcal{C}_{sim}$	**99%**	60%	83%	94%
found in $\mathcal{C}_{dissim}$	**94%**	75%	81%	88%

Table 1. Results for $D_{H,\mathrm{W}}$, the global HD, the Partial HD (PHD) and for the Sorted Values Vector of the LDMap (SVV). The PHD depends on a parameter and the presented results for the PHD are the best obtained.

in C_{sim} and 50 in C_{dissim}.
Then, the test is done on a distinct set of 75 LDMaps of C_{sim} and 200 of C_{dissim}. The choice of the sets in each class is randomized. Finally, we compare the results obtained manually and automatically.

The efficiency of four measure methods has been tested: our method (based on the LDMap), the global HD, the partial HD and the LDMap Sorted Value Vector (SVV). The last method is based on a vector including all the values of the LDMap that are sorted. As a consequence, all the LDMap spatial information is lost. So the spatial-information contribution can be quantified by comparison with the LDMap results. The classification methods depend on the measure :

- For our method and the SVV: the entry is a high dimension input. So the chosen classifier is based on a SVM (with a polynomial kernel) with a leaning step.
- For the other methods, the input is a real number, so an empirical distribution is computed on a learning set for each class C_{sim} and C_{dissim}, and the classification is made with the maximum likelihood method.

The decision method is different whether the measure result is an image (case of the LDMap) or a number (case of the HD and its variations). In the first case, the classification method is a SVM. In the second case, an empirical distribution for each class C_{sim} and C_{dissim} is computed from the learning set. The maximum likelihood method is used for the classification.

Results are summarized in table 1. As the PHD depends on a parameter, only the best results are presented in the table. The results show the efficiency of the local distance map both concerning spatial information (comparison with the SVV) and the ability of the local HD to catch the local dissimilarities (comparison with the global HD and the partial HD).

3 Conclusion

A new comparison method is presented that enables the local measure of the dissimilarities in the case of binary images. It based on a the Hausdorff distance that has been adapted to be windowed. The result is a distance map that give local dissimilarities measure and spatial distribution. This map can be fast computed and allows a generalization to gray-level images. Combined to a SVM classifier, this map give better classification rates than the other tested method.

Our aim is now to test it on a bigger database and to integrate it in a high-level method so as to exploit its properties.

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