

Hausdorff distance based multiresolution maps applied to an image similarity measure

E. Baudrier¹, G. Millon¹, F. Nicolier¹, R. Seulin², S. Ruan¹

¹ CReSTIC – URCA, IUT,
9, rue de Québec, 10026 Troyes Cedex, France
E-mail: {[e.baudrier](mailto:e.baudrier@iut-troyes.univ-reims.fr), [g.millon](mailto:g.millon@iut-troyes.univ-reims.fr), [f.nicolier](mailto:f.nicolier@iut-troyes.univ-reims.fr), [s.ruan](mailto:s.ruan@iut-troyes.univ-reims.fr)}@iut-troyes.univ-reims.
² Le2i – CNRS UMR 5158,
University of Burgundy – IUT,
12, rue de la fonderie, 71200 Le Creusot, France

ABSTRACT

In this paper, a new dissimilarity measure is presented. In the context of Content Based Image Retrieval (CBIR) applied on binary images, the dissimilarity measure, based on the Hausdorff distance applied locally through a sliding window, results in a distance map. Computed at different resolutions thanks to a multiresolution, the distance maps allow to refine the decision in a coarse-to-fine process. The decision is based on the comparison of the distance map histogram to reference histograms of the class of distance maps. As an application, a database of digitalized ancient illustrations is successfully processed by the introduced method.

Keywords: CBIR, dissimilarity measure, multiresolution.

1. INTRODUCTION

Image retrieval is an active domain. Retrieving images by their content, as opposed to meta-data, has become an important activity. It is classically composed of two stages: firstly, an information mining, which results in an image signature and secondly, a signature distance measure that is used to decide on the image similarity. In this process, the signature must capture conspicuous features in order to be as discriminating as possible in some user-defined sense. In general, the signature contains colour, shape or texture information¹. But the choice of the signature attributes is not easy and depends on the treated images. In the scope of binary images, we propose to replace this awkward information mining by a straight image comparison based on a modified Hausdorff distance (HD)^{2, 3} producing a distance map. The second step is then replaced by a decision process based on the distance map. While an information mining requires a priori knowledge on discriminating features before comparing the images, our process first expresses dissimilarities from the image comparison before taking a decision. This process developed for binary images is adaptable to pattern recognition. In this paper, we present the different stages of the measure process: firstly a morphological multiresolution analysis, secondly the construction of a HD map between two images. Then, a decision on the similarity of the images based on the distance map is presented at the end of section 2. Finally, we expose some results to show the efficiency of our method.

2. MULTIREOLUTION

Human dissimilarity measure can be viewed as a coarse-to-fine process. As a consequence, in a first approximation, a dissimilarity measure can be carried on a low resolution, which allows in addition to save computation time. This can be done thanks to a Multiresolution Analysis (MRA). Nevertheless, the scale-space operator should satisfy conditions to preserve the binary image main features. Many classical scale-space operators use Gaussian functions. They have good scale-space properties but they smooth transitions, which results in a loss in the binary information and could produce errors in low resolutions. This drawback is a common property of linear filters. On the contrary, nonlinear filters can avoid this problem. Among them, morphological operators are good candidates⁵. We have determined three criteria to choose the morphological MRA operator ϕ :

- should be edge-preserving,
- should be auto dual (i.e. ϕ preserves the black-to-white pixel ratio),
- should preserve the main features.

Obviously, the last criterion is subjective and is satisfied a posteriori. This led us to choose the morphological so-called median operator which fulfils these conditions⁶. The morphological MRA is thus described by the following process:

1. Non-linear median filtering (on a 2x2 window) of the approximation a_j ,
2. Down-sampling by a factor 2, giving a_{j-1} and details
3. Repeat the process up to scale J.

Even if this study focuses on approximations, details may be exploited and are obtained by the following formulae:

$$D_v(i,j) = | I(2i-1,2j-1) - I(2i-1,2j) | \quad (1a)$$

$$D_d(i,j) = | I(2i-1,2j-1) - I(2i,2j) | \quad (1b)$$

$$D_h(i,j) = | I(2i-1,2j-1) - I(2i,2j-1) | \quad (1c)$$

where $I(2i-1,2j-1)$, $I(2i-1,2j)$, $I(2i,2j-1)$ and $I(2i,2j)$ stand for the four pixels in the 2x2 window of the filter.



binarised original image
512x512 (resolution 0)



resolution 2 (128x128)



resolution 4 (32x32)

Figure 1. An example of three resolutions of the same picture.

3. DISTANCE MAP

3.1 Hausdorff distance

The HD has often been used in the content-based retrieval domain. Originally meant as a measure between two sets of points A and B in a metric space (E, d) , it can be viewed as a dissimilarity measure between two binary images A and B considering A and B , respectively the black pixels finite set of points of A and B . The definition of the HD is following

Given two finite sets of points $A=(a_1, \dots, a_n)$ and $B=(b_1, \dots, b_m)$, and an underlying distance d , the HD is given by

$$D_H(A,B)=\max(h(A,B),h(B,A)) \quad (2.1)$$

Where

$$h(\mathbf{A}, \mathbf{B}) = \max_{a \in \mathbf{A}} (\min_{b \in \mathbf{B}} (d(a, b))) \quad (2.2),$$

$h(\mathbf{A}, \mathbf{B})$ is the so-called *directed Hausdorff distance* and in this paper, the underlying distance $d(a, b)$ will be the L_∞ norm $d(a, b) = \|b - a\|_\infty$. For images, we use the same notation: $D_H(\mathbf{A}, \mathbf{B}) = D_H(\mathbf{A}, \mathbf{B})$. The interest of this measure comes first from its metric properties (in our application, on the space of finite set of points): non-negativity, identity, symmetry and triangle inequality. These properties correspond to our intuition for shape resemblance. Another source of interest is the following property:

Property 1

Let \mathbf{v} be a vector of \mathbf{R}^2 , $T_{\mathbf{v}}$ translation of vector \mathbf{v} and \mathbf{A} a finite set of points, then $d(\mathbf{A}, T_{\mathbf{v}}\mathbf{A}) = \|\mathbf{v}\|$. It implies that the Hausdorff distance matches our intuition in case of translation. Nevertheless, it measures the most mismatched points between \mathbf{A} and \mathbf{B} , which is not convenient. There is an extension that avoids this danger, the so-called partial Hausdorff distance, based on the following definition of directed distance:

Definition 2

$h_k(\mathbf{A}, \mathbf{B}) = k^{th} \min_{a \in \mathbf{A}} (d(a, b))$ where $k^{th}_{x \in X} f(x)$ denotes the k -th ranked value over the set X . This is no longer a metric but it does not take into account the most distant points which can be irrelevant for the measure. Other modifications have been developed to improve the robustness of the HD³.

3.2 The Hausdorff distance map

The main inconvenience of these Hausdorff distances lies in the fact that they give a global dissimilarity measure over images. To retrieve different images standing for the same scene, the image comparison has to be based on the measure of local differences. To do so, a new dissimilarity measure based on the HD is introduced and designated by $D_{H,W}$. It consists of making a local measure using a sliding-window W . At each sample point, the HD is computed on the part of the images viewed through W . Thus, from two binary images to be compared, a distance map M is computed. M depends on the parameters (w_x, w_y) , the size of the window W and $p=(p_h, p_v)$, the step between the sample points.

illustration of the distance map

The following figure gives examples of distance maps (DM) between image 1 and images 2,3 and 4. Images 1 and 2 are very similar and they result in a DM with low distance values. Image 3 illustrates the same scene than image 1 but the grass and the helms have been represented differently. In their DM, the high values are confined to these parts. Image 3 is completely different from image 1 and in their DM the high values are randomly distributed.

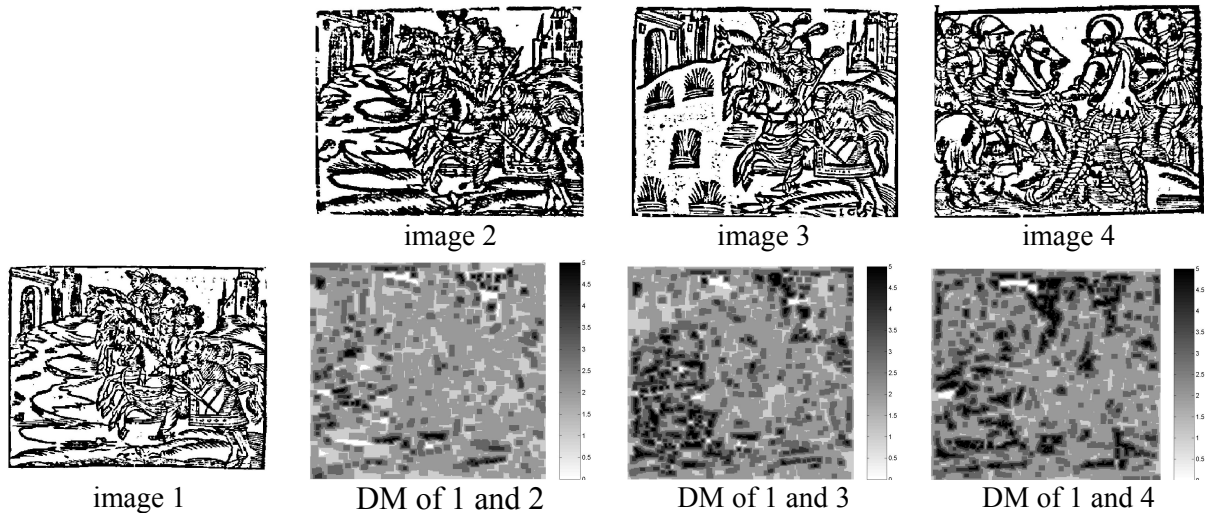


Fig 1. some pictures and their distance maps

Illustration of the sliding-window size

It is possible to take different sizes for the sliding-window. For a given resolution, the larger the sliding-window, the coarser the measured dissimilarities. An illustration is given below (Fig 2).

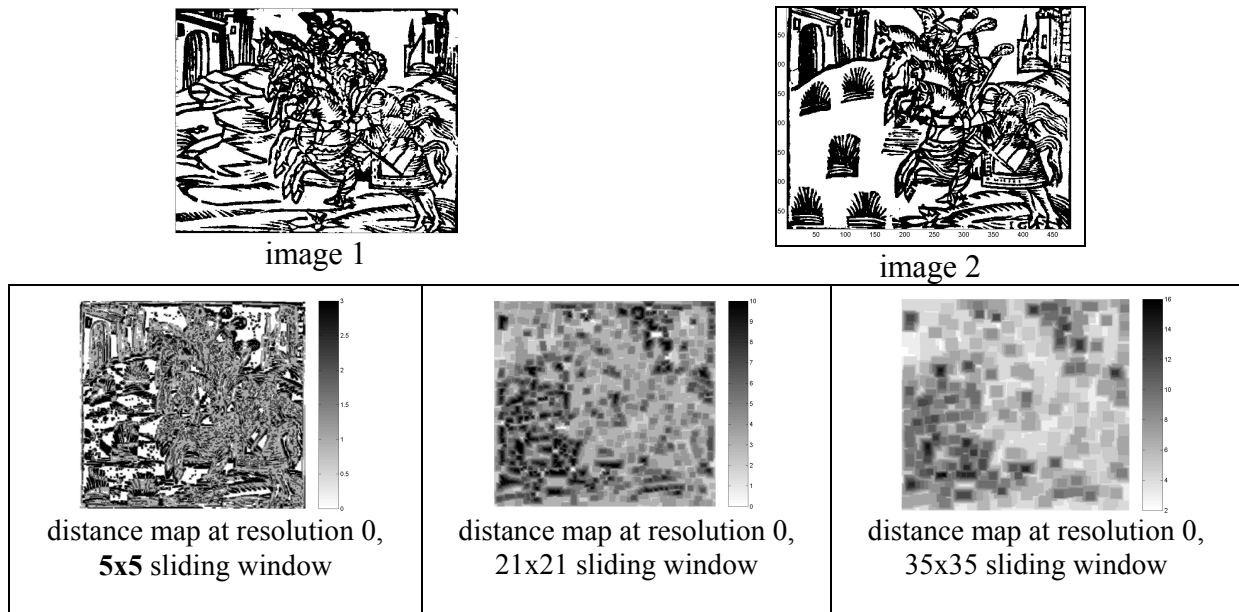


Figure 2. A 5x5 sliding-window shows fine dissimilarities, unlike a 35x35 one that overtones the coarse ones (the grass and the helmets).

Thus a variation in the window size enables to highlight different sizes of dissimilarities. As the human comparison process is a coarse-to-fine process, it is interesting to begin the comparison with a large sliding-window. Nevertheless, it is time consuming. Another way of using this property is to keep the same sliding-window at different resolutions.

Interest of the multiresolution

For a given sliding-window size, the finer the resolution, the more accurate the measured dissimilarities (see fig 3). In the coarse-to-fine process, the sliding-window size is constant across scales, thus more and more precise details of the images are compared.

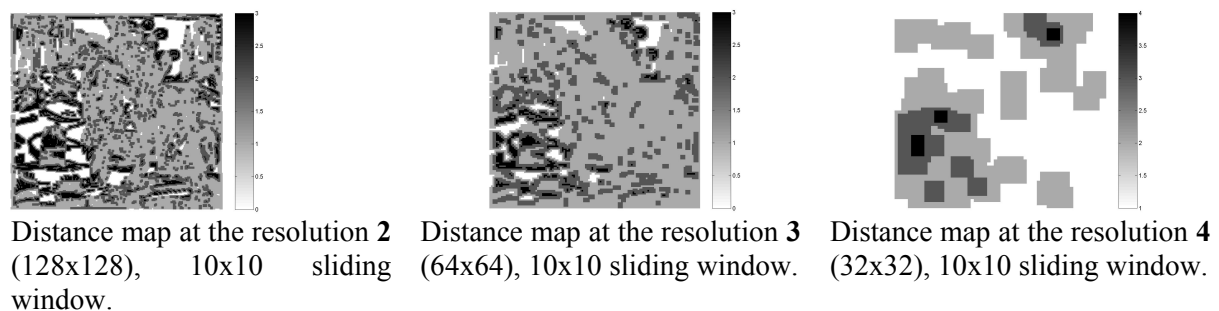


Figure 3. The distance maps of images 1 and 2. The resolution 2 shows dissimilarities even in the riders and their mounts. In resolution 4 remain only the coarse dissimilarities (the grass and the helmets).

4. DECISION PROCESS

4.1 distance map classification

The comparison process results in a distance map. It can be classified in two classes: C_{sim} that includes distance maps comparing similar images and C_{dissim} for those comparing dissimilar images. The classification is based on the comparison of the distance-map histogram to class reference histograms. The decision is then made by a chi 2 test.

4.2 The refinement stage

The comparison begins at a coarse resolution and results in a decision. But in the case of ambiguity in the decision (e.g. if membership probabilities of the distance map to the two classes are quite the same) then the comparison is done with a finer resolution which contains information on the dissimilarity in finer details. This can be done up to the original resolution (resolution 0). Practically, the comparison is firstly made at resolution 3 and the loop on the resolutions stops at resolution 1. Figure 3 represents the algorithm of the global process.

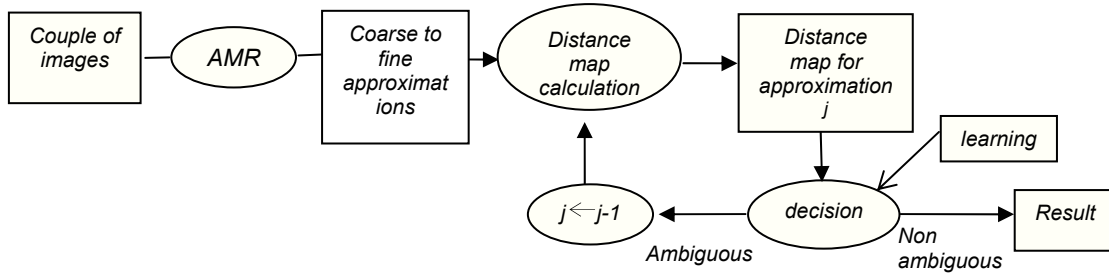


Figure 3. Global process algorithm

5. RESULTS

The method is tested on a database of digitalized ancient illustrations provided by the Troyes' library within the framework of the project ANITA⁷. These images, originally printed, have strong contrast which allows to binarize them with almost no loss. This database is composed of 68 images, some of them illustrating the same scene. They produce 2380 distance maps 103 of which belonging to C_{sim} . Our objective is to test the method's efficiency in retrieving similar images. We first compute the histogram average on a learning sample and then test the method on 94 image comparisons. Firstly, we sort them manually in the two classes C_{sim} (44 items) and C_{dissim} (50 items) introduced in the former paragraph. Secondly, we apply three classification methods. Finally, we compare the results obtained manually and automatically. The three classifications are the following: our method $D_{H,W}$; another one based on the global HD and the one - referred as H_{SD} - using the distance map, but with the simple difference locally instead of the HD. The results are summarized in the following table:

Table 1. Comparative results

Successful retrieval	$D_{H,W}$	HD	H_{SD}
In C_{sim}	95%	46%	66%
In C_{dif}	84%	70%	78%

In this experiment, the refinement stage has been used five times to reach a good result and never to obtain a wrong result.

6. CONCLUSION

In this paper, we have presented a method that allows to keep the measure properties of the Hausdorff distance and to suppress one of its inconveniences, the outlier sensibility. At the same time, the distance map produced has a different distribution whether the images are similar or dissimilar. This dissimilarity measure has been embedded in an AMR process, which results in a coarse-to-fine process. The good results of our method illustrate the interest of the distance map. Moreover, it enables the final user to find the dissimilarity zones between the compared images at one glance.

The perspectives of the study are to improve the distance map by making the size of the sliding-window automatically chosen and adaptative, and then to use a decision method taking in entry the whole distance map, so as to exploit the spatial information in it. Finally the improved method is planned to be applied to different databases so as to test its efficiency and robustness.

1. Arnold W. M. Smeulders, Marcel Worring, Simone Santini, Amarnath Gupta, and Ramesh Jain, "Content based image retrieval at the end of the early years", *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 22, no. 12, pp. 1349-1380, (2000).
2. Daniel P. Huttenlocher and William J. Rucklidge, "A multi-resolution technique for comparing images using the Hausdorff distance", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, pp. 705-706, (1993).
3. . Remco C. Veltkamp and Michiel Hagedoorn, "Shape similarity measures, properties, and constructions"
4. M. Dubuisson and A. K. Jain, "A modified Hausdorff distance for object matching", Proc. 12th International Conference on Pattern Recognition (ICPR), Jerusalem, Israel, (1994).
5. Corinne Vachier, "Morphological scale-space analysis and feature extraction", in *Proc. of international Conference of Image Processing (ICIP)*, (October 2001).
6. H.J.A.M. Heijmans and J. Goutsias, "Multiresolution signal decomposition schemes. Part 2: Linear and morphological pyramids", *IEEE Trans. Image Processing*, vol. 9, no. 11, pp. 1897-1913, (November 2000).
7. R. Seulin, Olivier Morel, G. Millon, and F. Nicolier, "Range image binarization: application to wooden stamps analysis", in *International Conference on Quality Control by Artificial Vision (QCAV) - IEEE*, Gatlinburg, Tennessee, USA, vol. 5132, pp. 252-258, SPIE, (may 2003).